Gaugino Mediation Combined with the Bulk Matter Randall-Sundrum Model

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Abstract

We investigate a simple 5D extension of the Minimal Supersymmetric (SUSY) Standard Model (SM) that is combined with the bulk matter Randall-Sundrum (RS) model, which gives a natural explanation the Yukawa coupling hierarchy. In this model, matter and gauge superfields reside in the 5D bulk while a SUSY breaking sector and the Higgs doublet superfields are localized on the infrared brane. The Yukawa coupling hierarchy in SM can be naturally explained through the wavefunction localization of the matter superfields. While sparticles obtain their flavor-blind soft SUSY breaking masses dominantly from the gaugino-mediated SUSY breaking, flavor-violating soft terms arise through the gravity-mediated SUSY breaking which are controlled by the wavefunction localization of the matter superfields. This structure of the model allows us to predict the sparticle mass spectrum including flavor-violating terms. We first explicitly determine the 5D disposition of matter superfields from the low energy experimental data on SM fermion masses, CKM matrix and the neutrino oscillation parameters. Then, we calculate particle mass spectra and estimate the effects of the flavor-violating soft terms, which should be compared with the current experimental constraints. With gravitino being the lightest sparticle (LSP), the next-to-LSP, which is long-lived, is predicted most likely to be either singlet smuon-like or selectron-like. The model can be tested at collider experiments through flavor-violating processes involving sparticles. The flavor structure among sparticle, once observed, gives us a clue to deep understanding of the origin of Yukawa coupling hierarchy.

1 Introduction

The gauge hierarchy problem has been the main motivation for physics beyond the Standard Model. One notable solution to this problem is offered by Randall-Sundrum (RS) model [1], which connects 4D Planck scale and the electroweak scale by means of 5D warped geometry. The RS setup also accommodates a natural explanation to the Yukawa coupling hierarchy in its extension with matters in the 5D bulk [2]. This bulk matter RS model solves the Yukawa hierarchy problem with the following common structure. We put Higgs on the infrared (IR) brane and fermions in the bulk. With non-hierarchical 5D Dirac masses, we localize the heavy fermions towards the IR brane and the light ones towards the ultraviolet (UV) brane. The wavefunction overlaps between the Higgs field and the fermion fields give rise to the Yukawa coupling hierarchy. In spite of their elegant solutions to the gauge hierarchy problem and the Yukawa hierarchy mystery, the RS models are under severe experimental constraints concerning the Kaluza-Klein (KK) scale due to the flavor-changing neutral currents caused by the KK states. Apart from the proton decay problem, the most stringent constraint comes from the data on $K^0-\bar{K}^0$ mixing and the 1st KK gluon mass should be $\gtrsim 21$ TeV [3]. This constraint thus spoils their solution to the gauge hierarchy problem as well as their experimental accessibility at the Large Hadron Collider (LHC). (However imposing some flavor symmetry softens the KK scale bound, see [4].)

In this paper, we study the supersymmterization of bulk matter RS model [5]. This model resorts to supersymmetry (SUSY) to solve the gauge hierarchy problem while maintaining the natural explanation of bulk matter RS model to the Yukawa hierarchy; the KK scale can be much higher than the electroweak scale and SUSY fills the gap between them. We especially investigate 5D Minimal SUSY Standard Model (MSSM) in RS spacetime, where we allow the gauge superfields to propagate in the bulk but localize Higgs superfields and the SUSY breaking sector on the IR brane, and the matter superfields are laid in the bulk with various 5D profiles.

All phenomenological SUSY models require a realistic SUSY breaking mechanism in harmony with experimental bounds. Our model naturally incorporates gaugino-mediated SUSY breaking mechanism as the matter superfields and SUSY breaking sector are (partly) separated by the 5th dimension while the gauge superfields couple to the SUSY breaking sector without suppression and mediate its effects to matter sector. At the same time, it explains the Yukawa coupling hierarchy through 5D localization of matter fields. Due to this structure, the sequestering is always incomplete; the superfields of 3rd generation particles lean towards the IR brane so that they couple to the SUSY breaking sector with less suppression. We thus have a unique pattern of flavor-violating soft mass terms that are related to the origin of the Yukawa hierarchy. Current experimental bounds on flavor violations give strong constraints on

the model and allow us to make predictions on the sparticle mass spectrum as well as on its flavor-violating effects. A similar setup was proposed in [6] as a 5D realization of "flavorful supersymmetry" [7].

Our model is a simple 5D extension of the MSSM in the RS spacetime, but has the ability to simultaneously provide a viable SUSY breaking mechanism and explain the hierarchical structure of Yukawa couplings. Since the sparticle mass spectrum and the Yukawa coupling hierarchy are rooted on the same 5D setup, this model has a strong predictive power on both of flavor-conserving and flavor-violating soft SUSY breaking terms.

In the next section, we write down a general form of MSSM in the bulk of RS spacetime equipped with gaugino-mediated SUSY breaking. In section 3, we assume that all couplings in 5D theory are of $O(1)$, and any hierarchical structure in 4D theory originates from 5D geometry. Based on this assumption, we determine 5D disposition of matter superfields from the observed fermion masses, Cabibbo-Kobayashi-Maskawa (CKM) matrix and neutrino oscillation data. In section 4, we make general remarks on the SUSY breaking mass spectrum. In section 5, we discuss the difference between our model and minimal flavor violation. In section 6, we calculate a sample of mass spectra and study experimental bounds on them. In section 7, we discuss a signature of the model, that is, unusual next-to-lightest sparticle (NLSP) and its flavor-violating decay. The last section is devoted for conclusion.

2 Setup

We consider 5D warped spacetime with the metric [1]:

$$\mathrm{d}s^2 \;=\; e^{-2k|y|}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}y^2 \;, \tag{1}$$

where y is the 5th dimension compactified on the orbifold $S^1/Z_2 : -\pi R \leq y \leq \pi R$, and k is the AdS curvature that is of the same order as the 5D Planck scale M_5. Assuming that the warp factor, $e^{-kR\pi}$, is much smaller than 1, we have the following relation for k and M_5 :

$$M_*^2 \;=\; \frac{M_5^3}{k}(1 - e^{-2kR\pi}) \;\simeq\; \frac{M_5^3}{k} \;, \tag{2}$$

where M_* is the 4D reduced Planck mass. This relation implies $k \sim M_5 \sim M_*$. We put a UV brane at $y = 0$ and an IR brane at $y = \pi R$. The fundamental scale on the UV brane is M_5, while that on the IR brane is $M_5 e^{-kR\pi}$. Note that in our model, $M_5 e^{-kR\pi}$ is not necessarily at TeV scale, but is at an intermediate scale between M_* and TeV.

All MSSM superfields reside in the bulk. However, for simplicity, the Higgs superfields are assumed to be localized on the IR brane. We adopt Polonyi model [8] for the SUSY breaking

sector and introduce a gauge-singlet chiral superfield X on the IR brane, whose F-component develops VEV to break supersymmetry. We take advantage of Giudice-Masiero mechanism [10], namely, we impose an appropriate R-symmetry to forbid the SUSY-conserving μ-term and force the μ-term to arise from SUSY breaking effects. In this paper, we assign the following R-charges to X and the MSSM superfields:

$$X : 0, \quad H_{u/d} : 0, \quad Q_i/U_i/D_i/L_i/E_i : +1,$$

where $H_u,\ H_d,\ Q_i,\ U_i,\ D_i,\ L_i,\ E_i$, respectively, denote the chiral superfields of up-type Higgs doublet, down-type Higgs doublet, SU(2) doublet quark, singlet up-type quark, singlet down-type quark, doublet lepton, singlet charged lepton. Note that the above assignment permits higher dimensional superpotential for light neutrino Majorana masses.

The 5D bulk action is described with 5D $\mathcal{N} = 1$ gauge multiplets and matter hypermultiplets. We use 4D superfield formalism extended with the 5th dimension y, following [5].

An off-shell 5D $\mathcal{N} = 1$ gauge multiplet consists of a 5D gauge field A_M ($M = 0, 1, 2, 3, 5$), two 4D Weyl spinors λ_1, λ_2, a real scalar Σ, a real auxiliary field D and a complex auxiliary field F, all of which transform as the adjoint representation of some gauge group. They are composed into one 4D $\mathcal{N} = 1$ gauge superfield V and one 4D $\mathcal{N} = 1$ chiral superfield χ that are

$$\begin{aligned} V &= -\theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \frac{1}{2}\bar{\theta}^2\theta^2 D\ , \\ \chi &= \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F\ . \end{aligned}$$

Under Z_2 parity: $y \to -y$, they transform as

$$V \ \to\ V\ , \qquad \chi\ \to\ -\chi\ .$$

The action for 5D $\mathcal{N} = 1$ gauge multiplets is given by

$$\begin{aligned} S_{5D\,gauge} \;=\; & \int \mathrm{d}y \int \mathrm{d}^4x\ e^{-4k|y|}\ \Bigg[\ \frac{1}{4(g_5^a)^2}\int \mathrm{d}^2\theta e^{k|y|}\ \mathrm{tr}\left\{\ (e^{\frac{3}{2}k|y|}W^{a\,\alpha})(e^{\frac{3}{2}k|y|}W^a_\alpha)\ +\ \mathrm{h.c.}\ \right\} \\ & + \frac{1}{(g_5^a)^2}\int \mathrm{d}^4\theta e^{2k|y|}\ \mathrm{tr}\left\{\ (\sqrt{2}\partial_y + \chi^{a\,\dagger})e^{-V}(-\sqrt{2}\partial_y + \chi^a)e^V\ -\ (\partial_y e^{-V})(\partial_y e^V)\right\}\Bigg]\ , \end{aligned} \tag{3}$$

where a labels gauge groups and $W^{a\,\alpha}$ denotes the field strength of V^a in 4D flat spacetime. When the unitary gauge, $A_5^a = 0$, is chosen, only V^a has a massless mode in 4D picture. This mode has no dependence on y and will be written as $V_0(x, \theta, \bar{\theta})$.

A 5D $\mathcal{N}=1$ hypermultiplet is expressed in terms of two 4D $\mathcal{N}=1$ chiral superfields Φ, Φ^c that are in conjugate representations of some gauge group. We assume that the former is Z_2-even and the latter Z_2-odd. Taking the basis of diagonal bulk mass, we have the following action for 5D $\mathcal{N}=1$ hyper-multiplets:

$$\begin{aligned} S_{5D\,chiral} &= \int \mathrm{d}y \int \mathrm{d}^4 x e^{-4k|y|} \left[\int \mathrm{d}^4\theta e^{2k|y|} \ (\Phi_i^\dagger e^{-V} \Phi_i \ + \ \Phi_i^c e^V \Phi_i^{c\,\dagger}) \right. \\ & \left. + \int \mathrm{d}^2\theta e^{k|y|} \ \Phi_i^c \{\partial_y - \chi/\sqrt{2} - (3/2 - c_i)k\} \Phi_i \ + \ \mathrm{h.c.} \right] , \end{aligned} \tag{4}$$

where i is a flavor index and c_i denotes the 5D bulk mass in unit of AdS curvature k. Only Φ_i has a massless mode in 4D picture, which will be written as $\phi_i(x,\theta)e^{(3/2-c_i)k|y|}$.

Let us write down the low-energy 4D effective action of the fields in the bulk, which is described with the massless modes of 5D $\mathcal{N}=1$ gauge multiplets and 5D $\mathcal{N}=1$ matter hyper-multiplets. After integrating over y, we obtain the following 4D effective action:

$$S_{4D\,eff.} = \int \mathrm{d}^4 x \left[\frac{2\pi R}{4 g_5^{a\,2}} \int \mathrm{d}^2\theta \ W^{a\alpha} W^a_\alpha \ + \ \mathrm{h.c.} + \int \mathrm{d}^4\theta \ 2 \frac{e^{(1-2c_i)kR\pi} - 1}{(1-2c_i)k} \ \phi_i^\dagger \ e^{-V} \phi_i \right] , \tag{5}$$

where the dimensionful 5D gauge coupling, g_5^a, is connected to 4D gauge coupling g_4^a by the relation: $g_5^a = \sqrt{2\pi R} g_4^a$.

Next we consider the theory on the IR brane. The IR scale, $M_5 e^{-kR\pi}$, is a free parameter of the model and is only assumed at an intermediate scale between the 5D Planck and the electroweak scales.

On the IR brane, we introduce Polonyi model for SUSY breaking:

$$S_{IR} \ \supset \ \int \mathrm{d}^4 x \left[\int \mathrm{d}^4\theta \ e^{-2kR\pi} \ (\ X^\dagger X \ + \ ... \ ) \ + \ \int \mathrm{d}^2\theta \ \mu_X^2 X \ + \ \mathrm{h.c.} \right] . \tag{6}$$

where the "..." term is for stabilizing the scalar potential of X at the origin. μ_X satisfies

$$e^{-2kR\pi} \mu_X^2 \ \sim \ M_5 e^{-kR\pi} \times \mathrm{TeV} \ , \tag{7}$$

which is equivalent to

$$\frac{\mu_X}{M_5} \ \sim \ \sqrt{\frac{\mathrm{TeV}}{M_5 e^{-kR\pi}}} \ , \tag{8}$$

so that it gives rise to gaugino masses at TeV scale through the VEV of F_X. Note that the scale of μ_X is between the 5D Planck and the IR scales. This scale is put in by hand, as in tree-level SUSY breaking models, or is generated through a dynamical SUSY breaking mechanism [9], of

which the Polonyi term (6) is the effective theory. We additionally assume that only the SUSY breaking term explicitly breaks the R-symmetry.

Other terms on the IR brane are listed below:

MSSM term:

$$
\begin{aligned}
S_{IR} \;\supset\; & \int d^4x \, \Bigg[\int d^4\theta \; e^{-2kR\pi} \; \Big\{ \; H_u^\dagger e^{-V} H_u \; + \; H_d^\dagger e^{-V} H_d \; \Big\} \\
& + \; \int d^2\theta \; e^{-3kR\pi} \; \Big\{ \; e^{(3-c_i-c_j)kR\pi} \; \frac{(y_u)_{ij}}{M_5} H_u U_i Q_j \; + \; e^{(3-c_k-c_l)kR\pi} \; \frac{(y_d)_{kl}}{M_5} H_d D_k Q_l \; \Big\} \; + \; \text{h.c.} \\
& + \; \int d^2\theta \; e^{-3kR\pi} \; e^{(3-c_m-c_n)kR\pi} \; \frac{(y_e)_{mn}}{M_5} H_d E_m L_n \; + \; \text{h.c.} \; \Bigg] \; . \qquad (9)
\end{aligned}
$$

Gaugino mass term:

$$
S_{IR} \;\supset\; \int d^4x \, \Big[\int d^2\theta \; d_a \frac{X}{M_5} W^{a\,\alpha} W^a_\alpha \; + \; \text{h.c.} \Big] \; . \qquad (10)
$$

Higgs SUSY breaking term:

$$
\begin{aligned}
S_{IR} \;\supset\; & \int d^4x \, \Bigg[\int d^4\theta \; e^{-2kR\pi} \; \Big\{ \; d_{mu} \frac{X^\dagger}{M_5} H_u H_d \; + \; d_{bmu} \frac{X^\dagger X}{M_5^2} H_u H_d \; + \; \text{h.c.} \; \Big\} \\
& + \; \int d^4\theta \; e^{-2kR\pi} \; \Big\{ \; d_{uA} \frac{X + X^\dagger}{M_5} H_u^\dagger H_u \; + \; d_{u0} \frac{X^\dagger X}{M_5^2} H_u^\dagger H_u \\
& \qquad + \; d_{dA} \frac{X + X^\dagger}{M_5} H_d^\dagger H_d \; + \; d_{d0} \frac{X^\dagger X}{M_5^2} H_d^\dagger H_d \; \Big\} \Bigg] \; . \qquad (11)
\end{aligned}
$$

Matter soft mass term:

$$
\begin{aligned}
S_{IR} \;\supset\; & \int d^4x \, \Bigg[\int d^4\theta \; e^{-2kR\pi} \; e^{(3-c_i-c_j)kR\pi} \; \Big\{ \; d_{Q1\,ij} \frac{X + X^\dagger}{M_5^2} \; Q_i^\dagger Q_j \; + \; d_{Q2\,ij} \frac{X^\dagger X}{M_5^3} \; Q_i^\dagger Q_j \; \Big\} \Bigg] \\
& + \; (\; Q \; \rightarrow \; U, \; D, \; L, \; E \;) \; . \qquad (12)
\end{aligned}
$$

A-term-generating term:

$$
\begin{aligned}
S_{IR} \;\supset\; & \int d^4x \, \Bigg[\int d^2\theta \; e^{-3kR\pi} \; \Big\{ \; e^{(3-c_i-c_j)kR\pi} \; \frac{(a_u)_{ij}}{M_5^2} X H_u U_i Q_j \; + \; e^{(3-c_k-c_l)kR\pi} \; \frac{(a_d)_{kl}}{M_5^2} X H_d D_k Q_l \\
& + \; e^{(3-c_m-c_n)kR\pi} \; \frac{(a_e)_{mn}}{M_5^2} X H_d E_m L_n \; \Big\} \; + \; \text{h.c.} \; \Bigg] \; . \qquad (13)
\end{aligned}
$$

We omitted brane kinetic terms because they only affect the overall normalization of the fields and are irrelevant to the point of our model.

We normalize $X, \; H_u, \; H_d, \; Q_i, \; U_i, \; D_i, \; L_i, \; E_i$ to make their kinetic terms of the 4D effective theory canonical. This is done by the following rescaling:

$$
\begin{aligned}
& X \; \rightarrow \; \tilde{X} = e^{-kR\pi} X, \quad H_u \; \rightarrow \; \tilde{H}_u = e^{-kR\pi} H_u, \quad H_d \; \rightarrow \; \tilde{H}_d = e^{-kR\pi} H_d, \\
& \qquad \phi_i \; \rightarrow \; \tilde{\phi}_i = \sqrt{2 \frac{e^{(1-2c_i)kR\pi} - 1}{(1-2c_i)k}} \; \phi_i, \qquad (14)
\end{aligned}
$$

where ϕ_i denotes $Q_i,\ U_i,\ D_i,\ L_i$ or E_i. Then the MSSM term becomes

$$\begin{aligned} S_{IR} \ &\supset \ \int \mathrm{d}^4x \ \left[\ \int \mathrm{d}^4\theta \ \left\{ \ \tilde{H}_u^\dagger e^{-V} \tilde{H}_u \ + \ \tilde{H}_d^\dagger e^{-V} \tilde{H}_d \ \right\} \right. \\ &+ \ \int \mathrm{d}^2\theta \ \left\{ \ \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} (y_u)_{ij} \ \tilde{H}_u \tilde{U}_i \tilde{Q}_j \right. \\ &+ \ \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}} \sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}} \frac{k}{M_5} (y_d)_{kl} \ \tilde{H}_d \tilde{D}_k \tilde{Q}_l \\ &+ \ \left. \left. \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}} \sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}} \frac{k}{M_5} (y_e)_{mn} \ \tilde{H}_d \tilde{E}_m \tilde{L}_n \ \right\} \ + \ \text{h.c.} \right] . \end{aligned} \tag{15}$$

The gaugino mass term will be

$$S_{IR} \ \supset \ \int \mathrm{d}^4x \ \left[\int \mathrm{d}^2\theta \ d_a \frac{\tilde{X}}{M_5 e^{-kR\pi}} W^{a\,\alpha} W^a_\alpha \ + \ \text{h.c.} \right] . \tag{16}$$

The Higgs SUSY breaking term will be

$$\begin{aligned} S_{IR} \ &\supset \ \int \mathrm{d}^4x \ \left[\int \mathrm{d}^4\theta \ \left\{ \ d_{mu} \frac{\tilde{X}^\dagger}{M_5 e^{-kR\pi}} \tilde{H}_u \tilde{H}_d \ + \ d_{bmu} \frac{\tilde{X}^\dagger \tilde{X}}{M_5^2 e^{-2kR\pi}} \tilde{H}_u \tilde{H}_d \ + \ \text{h.c.} \right. \right. \\ &+ \ \ d_{uA} \frac{\tilde{X} + \tilde{X}^\dagger}{M_5 e^{-kR\pi}} \tilde{H}_u^\dagger \tilde{H}_u \ + \ d_{u0} \frac{\tilde{X}^\dagger \tilde{X}}{M_5^2 e^{-2kR\pi}} \tilde{H}_u^\dagger \tilde{H}_u \\ &\qquad + \ d_{dA} \frac{\tilde{X} + \tilde{X}^\dagger}{M_5 e^{-kR\pi}} \tilde{H}_d^\dagger \tilde{H}_d \ + \ d_{d0} \frac{\tilde{X}^\dagger \tilde{X}}{M_5^2 e^{-2kR\pi}} \tilde{H}_d^\dagger \tilde{H}_d \ \left. \left. \right\} \right] . \end{aligned} \tag{17}$$

The matter soft mass term will be

$$\begin{aligned} S_{IR} \ &\supset \ \int \mathrm{d}^4x \ \left[\int \mathrm{d}^4\theta \ \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \ \frac{k}{M_5} \ \times \right. \\ &\qquad \left. \left\{ \ d_{Q1\,ij} \frac{\tilde{X} + \tilde{X}^\dagger}{M_5 e^{-kR\pi}} \ \tilde{Q}_i^\dagger \tilde{Q}_j \ + \ d_{Q2\,ij} \frac{\tilde{X}^\dagger \tilde{X}}{M_5^2 e^{-2kR\pi}} \ \tilde{Q}_i^\dagger \tilde{Q}_j \ \right\} \right] \\ &\quad + \ \left(\ \tilde{Q} \ \to \ \tilde{U}, \ \tilde{D}, \ \tilde{L}, \ \tilde{E} \ \right) . \end{aligned} \tag{18}$$

The A-term-generating term will be

$$\begin{aligned} S_{IR} \ &\supset \ \int \mathrm{d}^4x \ \left[\int \mathrm{d}^2\theta \ \left\{ \ \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} \frac{(a_u)_{ij}}{M_5 e^{-kR\pi}} \ \tilde{X} \tilde{H}_u \tilde{U}_i \tilde{Q}_j \right. \right. \\ &+ \ \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}} \sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}} \frac{k}{M_5} \frac{(a_d)_{kl}}{M_5 e^{-kR\pi}} \ \tilde{X} \tilde{H}_d \tilde{D}_k \tilde{Q}_l \\ &+ \ \left. \left. \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}} \sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}} \frac{k}{M_5} \frac{(a_e)_{mn}}{M_5 e^{-kR\pi}} \ \tilde{X} \tilde{H}_d \tilde{E}_m \tilde{L}_n \ \right\} \ + \ \text{h.c.} \ \right] . \end{aligned} \tag{19}$$

We introduce light neutrino masses by simply writing down higher dimensional operators on the IR brane, namely,

$$
\begin{aligned}
S_{IR} \supset & \int \mathrm{d}^4x \int \mathrm{d}^2\theta \; e^{-3kR\pi} \; e^{(3-c_p-c_q)kR\pi} \; (Y_\nu)_{pq} \frac{L_p H_u L_q H_u}{M_5} \; + \; \text{h.c.} \\
= & \int \mathrm{d}^4x \int \mathrm{d}^2\theta \sqrt{\frac{1-2c_p}{2\{1-e^{-(1-2c_p)kR\pi}\}}} \sqrt{\frac{1-2c_q}{2\{1-e^{-(1-2c_q)kR\pi}\}}} \; (Y_\nu)_{pq} \; \frac{\tilde{L}_p \tilde{H}_u \tilde{L}_q \tilde{H}_u}{M_5 e^{-kR\pi}} \; + \; \text{h.c.}
\end{aligned}
\tag{20}
$$

Note that the Kaluza-Klein (KK) scale, $M_5 e^{-kR\pi}$, is related to the scale of light neutrino masses. Another possibility is to introduce singlet neutrino superfields and adopt the seesaw mechanism [11]. In this case, the KK scale can be a free parameter of the model.

Now the MSSM Yukawa couplings are expressed as

$$
\begin{aligned}
(Y_u)_{ij} &= \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} (y_u)_{ij} \; , \\
(Y_d)_{kl} &= \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}} \sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}} \frac{k}{M_5} (y_d)_{kl} \; , \\
(Y_e)_{mn} &= \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}} \sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}} \frac{k}{M_5} (y_e)_{mn} \; ,
\end{aligned}
\tag{21}
$$

and the neutrino mass matrix m_ν is given by

$$
(m_\nu)_{pq} = \sqrt{\frac{1-2c_p}{2\{1-e^{-(1-2c_p)kR\pi}\}}} \sqrt{\frac{1-2c_q}{2\{1-e^{-(1-2c_q)kR\pi}\}}} \; (Y_\nu)_{pq} \; \frac{v_u^2}{M_5 e^{-kR\pi}} \; . \tag{22}
$$

The geometrical factor $\sqrt{(1-2c) \; / \; (2\{1-e^{-(1-2c)kR\pi}\})}$ has a unique property. For $c < 1/2$, it is approximated by $\sqrt{1/2-c}$ and is $O(1)$. For $c > 1/2$, it is approximated by $\sqrt{c-1/2}\, e^{-(c-1/2)kR\pi}$ and is exponentially suppressed. Therefore this factor can generate the large hierarchy of the Yukawa couplings without hierarchy. In the following, we assume that the components of 5D coupling matrices, y_u, y_d, y_e, Y_ν, are all $O(1)$ and that the hierarchical structure of MSSM Yukawa couplings and the neutrino mass matrix arises from the following terms:

$$
\sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \; .
$$

We define geometrical factors α_i as

$$
\alpha_i \equiv \sqrt{\frac{1-2c_{q\,i}}{2\{1-e^{-(1-2c_{q\,i})kR\pi}\}}} \qquad \text{with} \;\; i = 1, 2, 3 \tag{23}
$$

for the i-th generation of SU(2) doublet quark superfields. Similarly, we define $\beta_i,\ \gamma_i,\ \delta_i,\ \epsilon_i$ for SU(2) singlet up-type quark, singlet down-type quark, doublet lepton, singlet neutrino and singlet charged lepton, respectively. Thus, the up-type quark Yukawa matrix Y_u, the down-type Yukawa matrix Y_d and the charged lepton Yukawa matrix Y_e (in the basis of diagonal 5D bulk mass) are given by

$$(Y_u)_{ij} \sim \beta_i \alpha_j \ , \quad (Y_d)_{ij} \sim \gamma_i \alpha_j \ , \quad (Y_e)_{ij} \sim \epsilon_i \delta_j \ , \tag{24}$$

and the neutrino mass matrix m_ν is given by

$$(m_\nu)_{ij} \sim \delta_i \delta_j \, \frac{v_u^2}{M_5 e^{-kR\pi}}, \tag{25}$$

with VEV of the up-type Higgs doublet v_u.

3 Yukawa coupling hierarchy from geometry

In this section, we determine the order of the geometrical factors, $\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$, from the experimental data on SM fermion masses, CKM matrix and the neutrino oscillation parameters. Note that the geometrical factors must be evaluated at the KK scale, $ke^{-kR\pi}$, where the 5D theory is connected to the 4D effective theory. However, as is seen from [12], the renormalization group (RG) running changes the Yukawa couplings by at most a factor 2 and CKM matrix components by at most 1.2 through the RG running from $\sim 10^{15}$ GeV to electroweak scale. Also the neutrino mass matrix is affected only by $O(1)$ through the RG running [13]. Therefore we can estimate the order of $\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$ directly from the experimental data at low energies.

We first show the model's predictions on Yukawa eigenvalues and CKM matrix. Let us diagonalize the Yukawa matrices:

$$\begin{aligned} V_u Y_u U_u^\dagger &= \text{diag} \ , \\ V_d Y_d U_d^\dagger &= \text{diag} \ , \\ V_e Y_e U_e^\dagger &= \text{diag} \ . \end{aligned}$$

For successful diagonalization of the hierarchical Yukawa matrices, the unitary matrices, $U_u,\ U_d,\ V_u,\ V_d,\ U_e,\ V_e$, need to have the following structure:

$$U_u \sim U_d \sim \begin{pmatrix} 1 & 0 & 0 \\ \alpha_1/\alpha_2 & 1 & 0 \\ \alpha_1/\alpha_3 & \alpha_2/\alpha_3 & 1 \end{pmatrix} \ , \quad V_u \sim (\alpha \to \beta) \ , \quad V_d \sim (\alpha \to \gamma) \ ,$$
$$U_e \sim (\alpha \to \delta) \ , \quad V_e \sim (\alpha \to \epsilon) \tag{26}$$

which leads to

$$\begin{aligned} V_u Y_u U_u^\dagger &\sim \mathrm{diag}\ (\ \beta_1\alpha_1,\ \beta_2\alpha_2,\ \beta_3\alpha_3\ )\ , \\ V_d Y_d U_d^\dagger &\sim \mathrm{diag}\ (\ \gamma_1\alpha_1,\ \gamma_2\alpha_2,\ \gamma_3\alpha_3\ )\ , \\ V_e Y_e U_e^\dagger &\sim \mathrm{diag}\ (\ \epsilon_1\delta_1,\ \epsilon_2\delta_2,\ \epsilon_3\delta_3\ )\ . \end{aligned} \tag{27}$$

The hierarchical structure of CKM matrix U_{CKM} is given by

$$U_{CKM} \;=\; U_u U_d^\dagger \sim \begin{pmatrix} 1 & \alpha_1/\alpha_2 & \alpha_1/\alpha_3 \\ \alpha_1/\alpha_2 & 1 & \alpha_2/\alpha_3 \\ \alpha_1/\alpha_3 & \alpha_2/\alpha_3 & 1 \end{pmatrix} . \tag{28}$$

The absolute values of the CKM matrix components, $|U_{CKM}|$, at electroweak scale has been measured as [14]

$$\begin{aligned} &|U_{CKM}[M_W]| \\ = &\begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415 + 0.0010 - 0.0011 \\ 0.00874 + 0.00026 - 0.00037 & 0.0407 \pm 0.0010 & 0.999133 + 0.000044 - 0.000043 \end{pmatrix} . \end{aligned}$$

We approximate this matrix by the following formula:

$$|U_{CKM}| \;\simeq\; \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad \text{with} \quad \lambda = 0.22\ . \tag{29}$$

To discuss the neutrino mass matrix, we adopt the tri-bi-maximal mixing matrix [15] (which gives almost the best fit in the neutrino oscillation data):

$$U_{MNS} \;=\; \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

and the following data on neutrino mass squared differences [14]:

$$\Delta m_{21}^2 \;=\; 7.59 \pm 0.20 \times 10^{-5}\ \mathrm{eV}^2, \qquad |\Delta m_{32}^2| \;=\; 2.43 \pm 0.13 \times 10^{-3}\ \mathrm{eV}^2.$$

Also we assume that the mass of the lightest neutrino is negligible, for simplicity. Then the neutrino mass matrix, $U_{MNS}\ \mathrm{diag}(m_{\nu1}, m_{\nu2}, m_{\nu3}) U_{MNS}^\dagger$, is given by

$$U_{MNS}\ \mathrm{diag}(m_{\nu1}, m_{\nu2}, m_{\nu3}) U_{MNS}^\dagger \;=\; \begin{pmatrix} 0.29 & 0.29 & 0.29 \\ 0.29 & 2.8 & -2.2 \\ 0.29 & -2.2 & 2.8 \end{pmatrix} \times 10^{-11}\ \mathrm{GeV} \tag{30}$$

for the normal hierarchy case, while

$$U_{MNS}\ \mathrm{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) U_{MNS}^{\dagger} = \begin{pmatrix} 4.9 & 0.026 & 0.026 \\ 0.026 & 2.5 & 2.5 \\ 0.026 & 2.5 & 2.5 \end{pmatrix} \times 10^{-11}\ \mathrm{GeV} \tag{31}$$

for the inverted hierarchy case.

Now we are ready to compare the model parameters with the experimental data and estimate the order of $\alpha_i,\ \beta_i,\ \gamma_i,\ \delta_i,\ \epsilon_i$. For Yukawa eigenvalues, we simply have

$$\beta_1\alpha_1 \sim m_u/v\sin\beta\ , \quad \beta_2\alpha_2 \sim m_c/v\sin\beta\ , \quad \beta_3\alpha_3 \sim m_t/v\sin\beta\ , \tag{32}$$

$$\gamma_1\alpha_1 \sim m_d/v\cos\beta\ , \quad \gamma_2\alpha_2 \sim m_s/v\cos\beta\ , \quad \gamma_3\alpha_3 \sim m_b/v\cos\beta\ , \tag{33}$$

$$\epsilon_1\delta_1 \sim m_e/v\cos\beta\ , \quad \epsilon_2\delta_2 \sim m_\mu/v\cos\beta\ , \quad \epsilon_3\delta_3 \sim m_\tau/v\cos\beta\ . \tag{34}$$

Since the top Yukawa coupling is of ~ 1, we have $\alpha_3\beta_3 \sim 1$, which leads to

$$\alpha_3 \sim \beta_3 \sim 1\ . \tag{35}$$

Comparing (28) with (29), we find

$$\alpha_1 \sim \lambda^3\ , \quad \alpha_2 \sim \lambda^2. \tag{36}$$

We then have

$$\beta_1 \sim \lambda^{-3}\ m_u/v\sin\beta\ , \quad \beta_2 \sim \lambda^{-2}\ m_c/v\sin\beta\ , \tag{37}$$

$$\gamma_1 \sim \lambda^{-3}\ m_d/v\cos\beta\ , \quad \gamma_2 \sim \lambda^{-2}\ m_s/v\cos\beta\ , \quad \gamma_3 \sim m_b/v\cos\beta\ . \tag{38}$$

Next compare the matrix (25) with the observed neutrino mass matrix. For the normal hierarchy case, it is possible to reproduce the hierarchical structure of the neutrino mass matrix by adjusting

$$3\delta_1 \sim \delta_2 \sim \delta_3 \tag{39}$$

with the factor 3 coupling of the 5D theory. On the other hand, for the inverted hierarchy case, we cannot reproduce the neutrino mass matrix with $\mathcal{O}(1)$ couplings. The situation gets worse if we consider non-negligible mass of the lightest neutrino. Therefore, the model favors the normal hierarchy of neutrino masses with the relation (39). We estimate ϵ_i from the relation (39) as

$$\epsilon_1 \sim 3\ \delta_3^{-1}\ m_e/v\cos\beta\ , \quad \epsilon_2 \sim \delta_3^{-1}\ m_\mu/v\cos\beta\ , \quad \epsilon_3 \sim \delta_3^{-1}\ m_\tau/v\cos\beta\ . \tag{40}$$

Finally, we refer to the connection between the light neutrino mass scale and the KK scale. If the neutrino mass arises from higher dimensional superpotential, as in (25), the two scales are related through the following formula:

$$\delta_3^2 \, \frac{v_u^2}{M_5 e^{-kR\pi}} \sim 3 \times 10^{-11} \text{ GeV} \,. \tag{41}$$

Based on the relation above, we can estimate the KK scale from the value of δ_3.

4 Two origins of soft SUSY breaking terms

In this model, SUSY breaking terms have two origins. One is contact terms between the SUSY breaking sector and the MSSM sector on the IR brane (gravity mediation contributions) [16]. The other is radiative corrections, in particular, the renormalization group effects from gaugino soft masses (gaugino mediation contributions) [17]. For the superpartners of matter particles, the former induce flavor-violating soft terms while the latter mainly generate flavor-diagonal terms Due to the model's structure, the gravity mediation contributions are related to the 5D disposition of matter superfields that gives rise to the Yukawa coupling hierarchy.

As is argued in [18], when the square root of space-like momentum, $p \equiv \sqrt{-p^2}$, is larger than the KK scale, $ke^{-kR\pi}$, 5D gaugino propagator connecting the UV and the IR branes is suppressed by the factor

$$\exp[\, -p/(ke^{-kR\pi}) \,] \,.$$

Since the integral of loop momentum is done with Euclidized momentum, $l_E^2 = -l^2$, loop diagrams containing gaugino propagators between the two branes are also exponentially suppressed when the range of integral is limited to $[O(ke^{-kR\pi}),\ \infty)$, that is, when the renormalization scale is around the KK scale. Matter superfields confined on the UV brane receive SUSY breaking effects through loop diagrams involving gaugino propagators in the bulk and gaugino mass on the IR. Hence we argue that, at the scale of $ke^{-kR\pi}$, matter SUSY particles *in the bulk* gain SUSY breaking mass only through the contact terms on the IR brane, and radiative corrections through gauginos are negligible. When $p < ke^{-kR\pi}$, 5D gaugino propagator approaches to the 4D one divided by πR, and matter SUSY particles receive SUSY breaking effects through gaugino radiative corrections just as in 4D MSSM. Based on the discussions above, we calculate the SUSY breaking mass spectrum in the following way: At the renormalization scale $\mu_r = ke^{-kR\pi}$, SUSY breaking terms arise only from contact terms on the IR brane (gravity mediation). In particular, 1st generation matter sparticles that are localized towards the UV brane have almost zero soft mass. Below the scale of $ke^{-kR\pi}$, the RG equations of 4D MSSM controls the

mass spectrum (gaugino mediation). Therefore we can calculate the sparticle mass spectrum at the electroweak scale by solving the MSSM RG equations with the initial condition that, at $\mu_r = ke^{-kR\pi}$, SUSY breaking terms be given by the IR brane contact terms. In the rest of the paper, we denote the scale $ke^{-kR\pi}$ as M_{cut}.

At $\mu_r = M_{cut}$, the SUSY breaking terms are given as follows:

$$\text{gaugino masses} \quad M^a_{1/2} \;=\; -\, d_a\, 4(g^a_4)^2\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \tag{42}$$

$$\text{Higgs B}\mu \text{ term} \quad B\mu \;=\; d_{bmu}\, \frac{| < F_{\tilde{X}} > |^2}{M_5^2 e^{-2kR\pi}} \tag{43}$$

$$\text{Higgs soft masses} \quad m^2_{H_u} \;=\; (-d_{u0} + d^2_{uA})\, \frac{| < F_{\tilde{X}} > |^2}{M_5^2 e^{-2kR\pi}} \tag{44}$$

$$m^2_{H_d} \;=\; (-d_{d0} + d^2_{dA})\, \frac{| < F_{\tilde{X}} > |^2}{M_5^2 e^{-2kR\pi}} \tag{45}$$

$$\begin{aligned} \text{matter soft masses} \quad (m^2_Q)_{ij} &= (-d_{Q2ij} + d^2_{Q1ij})\, \alpha_i \alpha_j \frac{| < F_{\tilde{X}} > |^2}{M_5^2 e^{-2kR\pi}} \\ (\mathrm{Q},\ \alpha) &\to (\mathrm{U}, \beta),\ (\mathrm{D}, \gamma),\ (\mathrm{L}, \delta),\ (\mathrm{E}, \epsilon) \end{aligned} \tag{46}$$

$$\begin{aligned} \mathrm{A-terms} \quad (A_u)_{ij} &= -d_{uA}\, (y_u)_{ij}\, \beta_i \alpha_j\, \frac{k}{M_5}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \;+\; (a_u)_{ij}\, \beta_i \alpha_j\, \frac{k}{M_5}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \\ &= -d_{uA}\, (Y_u)_{ij}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \;+\; (a_u)_{ij}\, \beta_i \alpha_j\, \frac{k}{M_5}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \end{aligned} \tag{47}$$

$$(A_d)_{ij} \;=\; -d_{dA}\, (Y_d)_{ij}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \;+\; (a_d)_{ij}\, \gamma_i \alpha_j\, \frac{k}{M_5}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \tag{48}$$

$$(A_e)_{ij} \;=\; -d_{dA}\, (Y_e)_{ij}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \;+\; (a_e)_{ij}\, \epsilon_i \delta_j\, \frac{k}{M_5}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \tag{49}$$

where $\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$ are defined as in (23). In addition, the μ-term arises from the SUSY breaking effects (Giudice-Masiero mechanism):

$$\mu \;=\; d_{mu}\, \frac{< F_{\tilde{X}} >}{M_5 e^{-kR\pi}} \;. \tag{50}$$

Note that the flavor structure of matter soft masses and A-terms corresponds to the Yukawa coupling and neutrino mass matrix hierarchy in a unique way, governed by $\alpha_i,\ \beta_i,\ \gamma_i,\ \delta_i,\ \epsilon_i$.

We solve the MSSM RG equations from M_{cut} toward low energies with the initial conditions (42-50), and evaluate the sparticle mass spectrum at the electroweak scale.

Finally we remark on the nature of the lightest SUSY particle (LSP) and the next-to-lightest SUSY particle (NLSP) in this model. The gravitino mass is given by

$$m_{3/2} \;\simeq\; \frac{| < F_{\tilde{X}} > |}{\sqrt{3} M_*} \;=\; \frac{| < F_{\tilde{X}} > |}{\sqrt{3} M_5 e^{-kR\pi}}\, \frac{M_5 e^{-kR\pi}}{M_*} \;\sim\; \mathrm{TeV} \times e^{-kR\pi} \;, \tag{51}$$

and thus gravitino is always LSP, as in [19]. NLSP mainly consists of singlet sleptons whose flavor composition depends on the amount of gravity mediation contributions. Normally, the singlet stau is lighter than smuon and selectron due to its large Yukawa coupling, but in this model, it gains large soft mass through gravity mediation and may not be the lightest. As with other gravitino LSP scenarios, NLSP is long-lived because its coupling to gravitino is suppressed by $1/|< F_{\tilde{X}} >|$.

5 Comparison with Minimal Flavor Violation

The minimal flavor violation (MFV) is the setup that only SM Yukawa couplings violate flavor symmetry. In MFV, flavor-violating soft terms are generated via the MSSM RG equations involving Yukawa couplings. We here estimate the orders of the flavor-violating soft terms generated through RG running in MFV, and compare them with those via the gravity mediation in our model. We will see that the latter show different patterns from the former.

We first introduce a flavor basis where Y_u or Y_d and Y_e are diagonalized by the following unitary matrices U_*:

$$\begin{aligned} U_U Y_u U_{Qu} &= (diag.) \ , \\ U_D Y_d U_{Qd} &= (diag.) \ , \\ U_E Y_e U_L &= (diag.) \ . \end{aligned}$$

Note that U_*'s depend on the renormalization scale as Yukawa matrices receive RG corrections. We will estimate the orders of the changes of U_*'s through RG running. Below is the RG equations for Y_u:

$$\begin{aligned} \mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_U Y_u U_{Qu}) &= (\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_U)U_U^\dagger(U_U Y_u U_{Qu}) \ + \ U_U(\mu\frac{\mathrm{d}}{\mathrm{d}\mu}Y_u)U_{Qu} \ + \ (U_U Y_u U_{Qu})U_{Qu}^\dagger(\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_{Qu}) \\ &= (\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_U)U_U^\dagger(U_U Y_u U_{Qu}) \\ &+ \ \frac{1}{16\pi^2}\, U_U \ \{ \ Y_u Y_d^\dagger Y_d + 3Y_u Y_u^\dagger Y_u + 3\mathrm{tr}[Y_u^\dagger Y_u]Y_u + \mathrm{tr}[Y_D^\dagger Y_D]Y_u \\ &\quad - (\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2)Y_u \ \} \ U_{Qu} \\ &+ \ (U_U Y_u U_{Qu})U_{Qu}^\dagger(\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_{Qu}) \ , \end{aligned} \tag{52}$$

where Y_D is neutrino Dirac Yukawa coupling which appears when we introduce the singlet neutrinos lighter than M_{cut}. We hereafter adopt the GUT normalization for g_1. From (52), we see that $U_U Y_u U_{Qu}$ remains diagonal during RG running when the unitary matrices satisfy the

following conditions,

$$\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_U &= 0 \ , && (53)\\
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_{Qu} &= -\frac{1}{16\pi^2}\ (\mathrm{off}-\mathrm{diagonal\ components\ of\ } Y_d^\dagger Y_d)\ U_{Qu}\ . && (54)
\end{aligned}$$

In the same manner, we obtain the following conditions for keeping $U_D Y_d U_{Qd}$ and $U_E Y_e U_L$ diagonal:

$$\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_D &= 0 \ , && (55)\\
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_{Qd} &= -\frac{1}{16\pi^2}\ (\mathrm{off}-\mathrm{diagonal\ components\ of\ } Y_u^\dagger Y_u)\ U_{Qd}\ , && (56)\\
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_E &= 0 \ , && (57)\\
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}U_L &= -\frac{1}{16\pi^2}\ (\mathrm{off}-\mathrm{diagonal\ components\ of\ } Y_D^\dagger Y_D)\ U_L\ . && (58)
\end{aligned}$$

Now that we know how Y_u-diagonal basis, Y_d-diagonal basis and Y_e-diagonal basis change through RG running, we estimate the orders of MFV effects on A-terms in these bases. The MSSM RG equations for A-terms are given by:

$$\begin{aligned}
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}A_u &= 3A_uY_u^\dagger Y_u + 3Y_uY_u^\dagger A_u\\
&+ A_uY_d^\dagger Y_d + 2Y_uY_d^\dagger A_d\\
&+ 2(\ 3\mathrm{tr}[Y_u^\dagger A_u] - \frac{13}{15}g_1^2M_{1/2}^{a=1} - 3g_2^2M_{1/2}^{a=2} - \frac{16}{3}g_3^2M_{1/2}^{a=3}\ )Y_u\\
&+ (\ 3\mathrm{tr}[Y_u^\dagger Y_u] - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\ )A_u\\
&+ \mathrm{tr}[Y_D^\dagger Y_D]A_u + \mathrm{tr}[Y_D^\dagger A_D]Y_u\ , && (59)\\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}A_d &= 3A_dY_d^\dagger Y_d + 3Y_dY_d^\dagger A_d\\
&+ A_dY_u^\dagger Y_u + 2Y_dY_u^\dagger A_u\\
&+ 2(\ 3\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{7}{15}g_1^2M_{1/2}^{a=1} - 3g_2^2M_{1/2}^{a=2} - \frac{16}{3}g_3^2M_{1/2}^{a=3}\ )Y_d\\
&+ (\ 3\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\ )A_d\ , && (60)\\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}A_e &= 3A_eY_e^\dagger Y_e + 3Y_eY_e^\dagger A_e\\
&+ 2(\ 3\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{9}{5}g_1^2M_{1/2}^{a=1} - 3g_2^2M_{1/2}^{a=2}\ )Y_e\\
&+ (\ 3\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{9}{5}g_1^2 - 3g_2^2\ )A_e\\
&+ A_eY_D^\dagger Y_D + 2Y_eY_D^\dagger A_D\ , && (61)
\end{aligned}$$

where neutrino Dirac Yukawa coupling Y_D and the corresponding A-term A_D appear when we introduce singlet neutrinos lighter than M_{cut}. From (53-61), we obtain the following equations for A_u, A_d, A_e respectively in Y_u, Y_d, Y_e-bases:

$$
\begin{aligned}
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_U A_u U_{Qu}) &= 3U_U A_u Y_u^\dagger Y_u U_{Qu} + 3U_U Y_u Y_u^\dagger A_u U_{Qu} \\
&+ (U_U A_u U_{Qu})(\text{diagonal part of } U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu}) + 2U_U Y_u Y_d^\dagger A_d U_{Qu} \\
&+ 2\left(\ 3\mathrm{tr}[Y_u^\dagger A_u] - \frac{13}{15}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2} - \frac{16}{3}g_3^2 M_{1/2}^{a=3}\ \right)(U_U Y_u U_{Qu}) \\
&+ \left(\ 3\mathrm{tr}[Y_u^\dagger Y_u] - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\ \right)(U_U A_u U_{Qu}) \\
&+ \mathrm{tr}[Y_D^\dagger Y_D](U_U A_u U_{Qu}) + \mathrm{tr}[Y_D^\dagger A_D](U_U Y_u U_{Qu})\ , \qquad (62) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_D A_d U_{Qd}) &= 3U_D A_d Y_d^\dagger Y_d U_{Qd} + 3U_D Y_d Y_d^\dagger A_d U_{Qd} \\
&+ (U_D A_d U_{Qd})(\text{diagonal part of } U_{Qd}^\dagger Y_u^\dagger Y_u U_{Qd}) + 2U_D Y_d Y_u^\dagger A_u U_{Qd} \\
&+ 2\left(\ 3\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{7}{15}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2} - \frac{16}{3}g_3^2 M_{1/2}^{a=3}\ \right)(U_D Y_d U_{Qd}) \\
&+ \left(\ 3\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\ \right)(U_D A_d U_{Qd})\ , \qquad (63) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_E A_e U_L) &= 3U_E A_e Y_e^\dagger Y_e U_L + 3U_E Y_e Y_e^\dagger A_e U_L \\
&+ 2\left(\ 3\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{9}{5}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2}\ \right)(U_E Y_e U_L) \\
&+ \left(\ 3\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{9}{5}g_1^2 - 3g_2^2\ \right)(U_E A_e U_L) \\
&+ (U_E A_e U_L)(\text{diagonal part of } U_L^\dagger Y_D^\dagger Y_D U_L) + 2U_E Y_e Y_D^\dagger A_D U_L\ , \qquad (64)
\end{aligned}
$$

To study the effects of MFV, we set the initial conditions for A_u, A_d, A_e as

$$
(A_u)_{ij}|_{\text{ini.}} = M_u (Y_u)_{ij}\ , \qquad (A_d)_{ij}|_{\text{ini.}} = M_d (Y_d)_{ij}\ , \qquad (A_e)_{ij}|_{\text{ini.}} = M_e (Y_e)_{ij}
$$

with mass parameters, M_u, M_d and M_e. Then the terms $2U_U Y_u Y_d^\dagger A_d U_{Qu}$ in (62), $2U_D Y_d Y_u^\dagger A_u U_{Qd}$ in (63) and $2U_E Y_e Y_D^\dagger A_D U_L$ in (64) respectively give rise to off-diagonal terms of $(U_U A_u U_{Qu})$, $(U_D A_d U_{Qd})$, $(U_E A_e U_L)$, which were initially diagonal. These off-diagonal terms in turn generate off-diagonal terms through (62-64), but this does not change the orders of themselves. Noting that the orders of the Yukawa components in each basis are given as (δ_{ij} is the ordinary Kronecker's delta):

$$
\begin{aligned}
(U_U Y_u U_{Qu})_{ij} &\sim \beta_i \alpha_i\ \delta_{ij}\ , \qquad (U_U Y_d U_{Qu})_{ij} \sim \gamma_i \alpha_j\ , \\
(U_D Y_u U_{Qd})_{ij} &\sim \beta_i \alpha_j\ , \qquad (U_D Y_d U_{Qd})_{ij} \sim \gamma_i \alpha_i\ \delta_{ij}\ , \\
(U_E Y_e U_L)_{ij} &\sim \epsilon_i \delta_i\ \delta_{ij}\ , \qquad (U_E Y_D U_L)_{ij} \sim \zeta_i \delta_j\ ,
\end{aligned}
$$

where ζ_i's are the geometrical factors for singlet neutrinos satisfying $\zeta_i \lesssim 1$, we estimate the orders of the off-diagonal terms of $(U_U A_u U_{Qu})$, $(U_D A_d U_{Qd})$, $(U_E A_e U_L)$ that arise through RG running as $(i \neq j)$:

$$\begin{aligned}
\Delta(U_U A_u U_{Qu})_{ij} &\sim \frac{1}{16\pi^2} \ln(\frac{M_{cut}}{M_W})\ 2(U_U Y_u Y_d^\dagger A_d U_{Qu})_{ij} \\
&\sim \frac{2}{16\pi^2} \ln(\frac{M_{cut}}{M_W})\ \beta_i\alpha_i\ \alpha_i(\gamma_3)^2\alpha_j\ M_u\ , \qquad (65) \\
\Delta(U_D A_d U_{Qd})_{ij} &\sim \frac{1}{16\pi^2} \ln(\frac{M_{cut}}{M_W})\ 2(U_D Y_d Y_u^\dagger A_u U_{Qd})_{ij} \\
&\sim \frac{2}{16\pi^2} \ln(\frac{M_{cut}}{M_W})\ \gamma_i\alpha_i\ \alpha_i(\beta_3)^2\alpha_j\ M_d\ , \qquad (66) \\
\Delta(U_E A_e U_L)_{ij} &\sim \frac{1}{16\pi^2} \ln(\frac{M_{cut}}{M_{seesaw}})\ 2(U_E Y_e Y_D^\dagger A_D U_L)_{ij} \\
&\sim \frac{2}{16\pi^2} \ln(\frac{M_{cut}}{M_{seesaw}})\ \epsilon_i\delta_i\ \delta_i(\zeta_3)^2\delta_j\ M_e\ , \qquad (67)
\end{aligned}$$

where M_{seesaw} indicates the mass scale of the singlet neutrinos if they exist. We used the approximation that $\sum_k(\gamma_k)^2 \simeq (\gamma_3)^2$, $\sum_k(\beta_k)^2 \simeq (\beta_3)^2$ and $\sum_k(\zeta_k)^2 \simeq (\zeta_3)^2$. As for diagonal terms of $(U_U A_u U_{Qu})$, $(U_D A_d U_{Qd})$, $(U_E A_e U_L)$, the equations (65-67) do not change their orders. In conclusion, the orders of MFV effects on A-terms are given by the estimates (65-67).

We next estimate the orders of MFV effects on matter soft mass terms, m_Q^2, m_U^2, m_D^2, m_L^2, m_E^2, in the basis where Y_u or Y_d and Y_e are diagonal. Below is the list of those terms in MSSM

RG equations that give rise to flavor non-universal soft masses:

$$
\begin{aligned}
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_Q^2 &\supset Y_u^\dagger Y_u m_Q^2 + m_Q^2 Y_u^\dagger Y_u + 2Y_u^\dagger m_U^2 Y_u + 2(Y_u^\dagger Y_u)m_{H_u}^2 \\
&+ Y_d^\dagger Y_d m_Q^2 + m_Q^2 Y_d^\dagger Y_d + 2Y_d^\dagger m_D^2 Y_d + 2(Y_d^\dagger Y_d)m_{H_d}^2 \\
&+ 2A_u^\dagger A_u + 2A_d^\dagger A_d \ , \qquad (68) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_U^2 &\supset 2Y_u Y_u^\dagger m_U^2 + 2m_U^2 Y_u Y_u^\dagger + 4Y_u m_Q^2 Y_u^\dagger + 4(Y_u Y_u^\dagger)m_{H_u}^2 \\
&+ 4A_u A_u^\dagger \ , \qquad (69) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_D^2 &\supset 2Y_d Y_d^\dagger m_D^2 + 2m_D^2 Y_d Y_d^\dagger + 4Y_d m_Q^2 Y_d^\dagger + 4(Y_d Y_d^\dagger)m_{H_d}^2 \\
&+ 4A_d A_d^\dagger \ , \qquad (70) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_L^2 &\supset Y_e^\dagger Y_e m_L^2 + m_L^2 Y_e^\dagger Y_e + 2Y_e^\dagger m_E^2 Y_e + 2(Y_e^\dagger Y_e)m_{H_d}^2 \\
&+ 2A_e^\dagger A_e \\
&+ Y_D^\dagger Y_D m_L^2 + m_L^2 Y_D^\dagger Y_D + 2Y_D^\dagger m_N^2 Y_D + 2(Y_D^\dagger Y_D)m_{H_u}^2 + 2A_D^\dagger A_D \ , \qquad (71) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_E^2 &\supset 2Y_e Y_e^\dagger m_E^2 + 2m_E^2 Y_e Y_e^\dagger + 4Y_e m_L^2 Y_e^\dagger + 4(Y_e Y_e^\dagger)m_{H_d}^2 \\
&+ 4A_e A_e^\dagger \ , \qquad (72)
\end{aligned}
$$

where again, Y_D and A_D appear when singlet neutrinos lighter than M_{cut} exist. From (53-58) and (68-72), we obtain the following equations for m_Q^2 in Y_u-diagonal basis, m_U^2 in Y_u-diagonal

basis, m_D^2 in Y_d-diagonal basis, m_L^2 in Y_e-diagonal basis and m_E^2 in Y_e-diagonal basis:

$$\begin{aligned}
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_{Qu}^\dagger m_Q^2 U_{Qu}) &\supset U_{Qu}^\dagger Y_u^\dagger Y_u m_Q^2 U_{Qu} + U_{Qu}^\dagger m_Q^2 Y_u^\dagger Y_u U_{Qu} \\
&+ 2U_{Qu}^\dagger Y_u^\dagger m_U^2 Y_u U_{Qu} + 2(U_{Qu}^\dagger Y_u^\dagger Y_u U_{Qu}) m_{H_u}^2 \\
&+ U_{Qu}^\dagger(\text{diagonal parts of } Y_d^\dagger Y_d) U_{Qu} (U_{Qu}^\dagger m_Q^2 U_{Qu}) \\
&+ (U_{Qu}^\dagger m_Q^2 U_{Qu}) U_{Qu}^\dagger(\text{diagonal parts of } Y_d^\dagger Y_d) U_{Qu} \\
&+ 2U_{Qu}^\dagger Y_d^\dagger m_D^2 Y_d U_{Qu} + 2(U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu}) m_{H_d}^2 \\
&+ 2U_{Qu}^\dagger A_u^\dagger A_u U_{Qu} + 2U_{Qu}^\dagger A_d^\dagger A_d U_{Qu} \ , \qquad (73) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_U m_U^2 U_U^\dagger) &\supset 2U_U Y_u Y_u^\dagger m_U^2 U_U^\dagger + 2U_U m_U^2 Y_u Y_u^\dagger U_U^\dagger \\
&+ 4U_U Y_u m_Q^2 Y_u^\dagger U_U^\dagger + 4(U_U Y_u Y_u^\dagger U_U^\dagger) m_{H_u}^2 \\
&+ 4U_U A_u A_u^\dagger U_U^\dagger \ , \qquad (74) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_D m_D^2 U_D^\dagger) &\supset 2U_D Y_d Y_d^\dagger m_D^2 U_D^\dagger + 2U_D m_D^2 Y_d Y_d^\dagger U_D^\dagger \\
&+ 4U_D Y_d m_Q^2 Y_d^\dagger U_D^\dagger + 4(U_D Y_d Y_d^\dagger U_D^\dagger) m_{H_d}^2 \\
&+ 4U_D A_d A_d^\dagger U_D^\dagger \ , \qquad (75) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_L^\dagger m_L^2 U_L) &\supset U_L^\dagger Y_e^\dagger Y_e m_L^2 U_L + U_L^\dagger m_L^2 Y_e^\dagger Y_e U_L \\
&+ 2U_L^\dagger Y_e^\dagger m_E^2 Y_e U_L + 2(U_L^\dagger Y_e^\dagger Y_e U_L) m_{H_d}^2 \\
&+ 2U_L^\dagger A_e^\dagger A_e U_L \\
&+ U_L^\dagger(\text{diagonal parts of } Y_D^\dagger Y_D) U_L (U_L^\dagger m_L^2 U_L) \\
&+ (U_L^\dagger m_L^2 U_L) U_L^\dagger(\text{diagonal parts of } Y_D^\dagger Y_D) U_L \\
&+ 2U_L^\dagger Y_D^\dagger m_N^2 Y_D U_L + 2(U_L^\dagger Y_D^\dagger Y_D U_L) m_{H_u}^2 + 2U_L^\dagger A_D^\dagger A_D U_L \ , \qquad (76) \\
16\pi^2\mu\frac{\mathrm{d}}{\mathrm{d}\mu}(U_E m_E^2 U_E^\dagger) &\supset 2U_E Y_e Y_e^\dagger m_E^2 U_E^\dagger + 2U_E m_E^2 Y_e Y_e^\dagger U_E^\dagger \\
&+ 4U_E Y_e m_L^2 Y_e^\dagger U_E^\dagger + 4(U_E Y_e Y_e^\dagger U_E^\dagger) m_{H_d}^2 \\
&+ 4U_E A_e A_e^\dagger U_E^\dagger \ . \qquad (77)
\end{aligned}$$

To extract the effects of MFV on matter soft mass terms, we consider the case with the following initial conditions:

$$\begin{aligned}
(m_*^2)_{ij}|_{\text{ini.}} &= m_{*0}^2\delta_{ij} \quad (* = Q, U, D, L, E, N) \ , \\
(A_u)_{ij}|_{\text{ini.}} &= M_u(Y_u)_{ij} \ , \qquad (A_d)_{ij}|_{\text{ini.}} = M_d(Y_d)_{ij} \ , \qquad (A_e)_{ij}|_{\text{ini.}} = M_e(Y_e)_{ij} \ , \\
(A_D)_{ij}|_{\text{ini.}} &= M_D(Y_D)_{ij} \ .
\end{aligned}$$

In (71), the terms involving Y_d or A_d induce the off-diagonal terms of $(U_{Qu}^\dagger m_Q^2 U_{Qu})$ of the order

$(i \neq j)$:

$$\Delta(U_{Qu}^\dagger m_Q^2 U_{Qu})_{ij} \;\; \sim \;\; \frac{1}{16\pi^2} \; \ln(\frac{M_{cut}}{M_W}) \; \alpha_i(\gamma_3)^2\alpha_j \; \{ \; 2m_{Q0}^2 + 2m_{D0}^2 + 2m_{Hd}^2 + 2M_d^2 \; \} \; . \quad (78)$$

It is then clear that, in Y_d-diagonal basis, $(U_{Qd}^\dagger m_Q^2 U_{Qd})$ obtain the off-diagonal terms of the order $(i \neq j)$:

$$\Delta(U_{Qd}^\dagger m_Q^2 U_{Qd})_{ij} \;\; \sim \;\; \frac{1}{16\pi^2} \; \ln(\frac{M_{cut}}{M_W}) \; \alpha_i(\beta_3)^2\alpha_j \; \{ \; 2m_{Q0}^2 + 2m_{U0}^2 + 2m_{Hu}^2 + 2M_u^2 \; \} \; . \quad (79)$$

If singlet neutrinos lighter than M_{cut} exist, the terms in (74) involving Y_D or A_D induce the off-diagonal terms of $(U_L^\dagger m_L^2 U_L)$ of the order $(i \neq j)$:

$$\Delta(U_L^\dagger m_L^2 U_L)_{ij} \;\; \sim \;\; \frac{1}{16\pi^2} \; \ln(\frac{M_{cut}}{M_{seesaw}}) \; \delta_i(\zeta_3)^2\delta_j \; \{ \; 2m_{L0}^2 + 2m_{N0}^2 + 2m_{Hu}^2 + 2M_D^2 \; \} \; . \quad (80)$$

On the other hand, for $(U_U m_U^2 U_U^\dagger)$, off-diagonal terms arise from the following two terms in (74):

$$\begin{aligned} 4U_U Y_u m_Q^2 Y_u^\dagger U_U^\dagger &= 4(U_U Y_u U_{Qu}^\dagger)(U_{Qu} m_Q^2 U_{Qu}^\dagger)(U_{Qu}^\dagger Y_u^\dagger U_U^\dagger) \; , \\ 4U_U A_u A_u^\dagger U_U^\dagger &= 4(U_U A_u U_{Qu}^\dagger)(U_{Qu}^\dagger A_u^\dagger U_U^\dagger) \; . \end{aligned} \quad (81)$$

Off-diagonal terms of $(U_{Qu}^\dagger m_Q^2 U_{Qu})$ induced through (73) and those of $(U_U A_u U_{Qu})$ induced through (62) in turn give rise to off-diagonal terms of $(U_U m_U^2 U_U^\dagger)$ through (81). Therefore, from (78) and (65), we estimate off-diagonal terms of $(U_U m_U^2 U_U^\dagger)$ induced by RG running as $(i \neq j)$:

$$\begin{aligned} \Delta(U_U m_U^2 U_U^\dagger)_{ij} \;\; &\sim \;\; \left[\frac{1}{16\pi^2}\ln(\frac{M_{cut}}{M_W})\right]^2 \\ &\times \;\; [\; 4\beta_i\alpha_i\alpha_i(\gamma_3)^2\alpha_j\alpha_j\beta_j(2m_{Q0}^2 + 2m_{D0}^2 + 2m_{Hd}^2 + 2M_d^2) \\ &+ \;\; 4\sum_k \; (\; \beta_i\alpha_i\delta_{ik}M_u + \beta_i\alpha_i\alpha_i(\gamma_3)^2\alpha_k M_u \;)(\; \alpha_k\beta_k\delta_{kj}M_u + \alpha_k(\gamma_3)^2\alpha_j\alpha_j\beta_j \;) \;] \\ &\sim \;\; \left[\frac{1}{16\pi^2}\ln(\frac{M_{cut}}{M_W})\right]^2 \\ &\times \;\; [\; 4\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j \; (2m_{Q0}^2 + 2m_{D0}^2 + 2m_{Hd}^2 + 2M_d^2) \\ &+ \;\; 8\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j M_u^2 \; + \; 4\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_3)^2(\gamma_3)^2(\alpha_j)^2\beta_j M_u^2 \;] \; . \end{aligned} \quad (82)$$

Likewise, we obtain the following estimates on off-diagonal terms of $(U_D m_D^2 U_D^\dagger)$ and $(U_E m_E^2 U_E^\dagger)$

$(i \neq j)$:

$$\begin{aligned}
\Delta(U_D m_D^2 U_D^\dagger)_{ij} &\sim \left[\frac{1}{16\pi^2}\ln(\frac{M_{cut}}{M_W})\right]^2 \\
&\times \ [\ 4\gamma_i(\alpha_i)^2(\beta_3)^2(\alpha_j)^2\gamma_j\ (2m_{Q0}^2+2m_{U0}^2+2m_{Hu}^2+2M_u^2) \\
&+ \ 8\gamma_i(\alpha_i)^2(\beta_3)^2(\alpha_j)^2\gamma_j M_d^2 \ + \ 4\gamma_i(\alpha_i)^2(\beta_3)^2(\alpha_3)^2(\beta_3)^2(\alpha_j)^2\gamma_j M_d^2\ ]\ , \quad (83) \\
\Delta(U_E m_E^2 U_E^\dagger)_{ij} &\sim \left[\frac{1}{16\pi^2}\ln(\frac{M_{cut}}{M_{seesaw}})\right]^2 \\
&\times \ [\ 4\epsilon_i(\delta_i)^2(\zeta_3)^2(\delta_j)^2\epsilon_j\ (2m_{L0}^2+2m_{N0}^2+2m_{Hu}^2+2M_D^2) \\
&+ \ 8\epsilon_i(\delta_i)^2(\zeta_3)^2(\delta_j)^2\epsilon_j M_d^2 \ + \ 4\epsilon_i(\delta_i)^2(\zeta_3)^2(\delta_3)^2(\zeta_3)^2(\delta_j)^2\epsilon_j M_e^2\ ]\ . \quad (84)
\end{aligned}$$

In summary, the orders of MFV effects on soft mass terms are given in (79, 80), (82-84).

We have estimated the orders of MFV effects on A-terms and soft mass terms in the basis where Y_u or Y_d and Y_e are diagonal. In the rest of the section, we compare MFV effects with flavor-violating gravity mediation effects of our model and discuss their difference.

Flavor-violating gravity mediation effects at the scale M_{cut} can be read from (46-49). We assume that the couplings in 5D theory, d_*, a_*, are all $O(1)$. This is a natural assumption because we are trying to explain the hierarchy of 4D theory from 5D geometrical point of view. In an arbitrary basis, the flavor-violating parts of A-terms that arise from gravity mediation and are not proportional to the corresponding Yukawa couplings are given by

$$(A_u)_{ij} \ \supset \ \sim \ \beta_i\alpha_j M_X\ , \quad (85)$$

$$(A_d)_{ij} \ \supset \ \sim \ \gamma_i\alpha_j M_X\ , \quad (86)$$

$$(A_e)_{ij} \ \supset \ \sim \ \epsilon_i\delta_j M_X \quad (87)$$

at the scale M_{cut}, where M_X is defined as

$$M_X \ \equiv \ \frac{|<F_{\tilde{X}}>|}{M_5 e^{-kR\pi}}\ .$$

Matter soft mass terms that arise from gravity mediation are given by

$$\begin{aligned}
(m_Q^2)_{ij} &\sim \ \alpha_i\alpha_j M_X^2\ , \\
(\mathrm{Q},\ \alpha) &\to \ (\mathrm{U},\beta),\ (\mathrm{D},\gamma),\ (\mathrm{L},\delta),\ (\mathrm{E},\epsilon) \quad (88)
\end{aligned}$$

at the scale M_{cut}. First, we argue that, at the electroweak scale, the flavor-violating parts of A-terms that are not proportional to the corresponding Yukawa couplings are still estimated as in (85-87) and flavor-violating parts of matter soft mass terms are estimated as in (88). This is understood from the form of RG equations; the right hand sides of the RG equations

(59-61) depend on the Yukawa couplings and A-terms themselves. For the component $(A_u)_{ij}$, the right hand side of (59) is at least proportional to $\beta_i\alpha_j$. Similarly, the right hand sides of (68-72), which express flavor-violating contributions, depend on the Yukawa couplings and A-terms. The flavor-violating part of the RG equation for the component $(m_Q^2)_{ij}$ is at least proportional to $\alpha_i\alpha_j$. The same discussion applies to other A-terms and matter soft masses, and we conclude that RG running keeps the orders of flavor-violating parts of A-terms and matter soft mass terms as in (85-88). Second, we argue that the estimates (85-88) are valid even in Y_u or Y_d and Y_e-diagonal basis. This is because the 5D couplings $(y_*)_{ij}$, $(a_*)_{ij}$, d_{*1ij} and d_{*2ij} in (46-49) are independent of each other. Therefore the matrices $(a_*)_{ij}$, d_{*1ij} and d_{*2ij}, which give rise to A-terms and matter soft masses, are arbitrary even when $(y_*)_{ij}$ is diagonal.

Let us compare the orders of MFV effects (65-67, 79, 80, 82-84) and those of flavor-violating gravity mediation effects (85-88). We assume that M_u, M_d, M_e in (65-67) and $m_{*0}^2, m_{Hu}^2, m_{Hd}^2$ in (79, 80, 82-84) are of the same order as M_X in (85-88). For the A-term components $(A_u)_{1j}$, $(A_u)_{2j}$ in Y_u-diagonal basis and $(A_d)_{1j}$, $(A_d)_{2j}$ in Y_d-diagonal basis, MFV effects are always much smaller than flavor-violating gravity mediation effects because the former are suppressed by $(\alpha_1)^2$ or $(\alpha_2)^2$ compared to the latter. For the components $(A_u)_{3j}, (A_d)_{3j}$, MFV effects can be of the same order as flavor-violating gravity mediation effects. For the A-term $(A_e)_{ij}$, MFV effects are much smaller than flavor-violating gravity mediation effects when the order of δ_3 is significantly smaller than 1 (we will see in the next section that this is the case for a realistic mass spectrum). If singlet neutrinos lighter than M_{cut} do *not* exist, A_e is diagonal.

For the soft mass terms m_Q^2 and m_L^2 (if singlet neutrinos lighter than M_{cut} exist), MFV effects can be of the same order as flavor-violating gravity mediation effects. For the terms m_U^2 in Y_u-diagonal basis and m_D^2 in Y_d-diagonal basis, MFV effects are always much smaller than flavor-violating gravity mediation effects except their (3,3)-components. This is because the components of these terms other than (3,3) are at least suppressed by $(\alpha_1)^2$ or $(\alpha_2)^2$. For the components $(m_U^2)_{33}$ in Y_u-diagonal basis and $(m_D^2)_{33}$ in Y_d-diagonal basis, the former can be as large as the latter. For the term m_E^2 in Y_e-diagonal basis, MFV effects are much smaller than flavor-violating gravity mediation effects when the order of δ_3 is significantly smaller than 1. If singlet neutrinos lighter than M_{cut} do *not* exist, there is no MFV on m_L^2, m_E^2.

In this section, we have discussed the difference between the flavor-violating soft terms in MFV and those generated by the gravity mediation of our model. We have proved that, for some components of A-terms and soft mass terms, the gravity mediation contribution dominates. Therefore, it is in principle possible to distinguish our model from other SUSY models with MFV.

6 Particle mass spectra and experimental constraints

We calculate a sample of mass spectra and check that this model provides a realistic mass spectrum consistent with current experimental bounds.

Our numerical analysis is done in the following way. We fix the cutoff scale, M_{cut}, which is of the same order as the KK scale, $M_5 e^{-kR\pi}$, from the relation (41)

$$M_{cut} \sim \delta_3^2 \frac{v_u^2}{3 \times 10^{-11}\ \mathrm{GeV}} \simeq \delta_3^2 \times 10^{15}\ \mathrm{GeV} .$$

We assume that contact term couplings between the MSSM fields and the SUSY breaking field in (42-50) are all $O(1)$ and adopt the following initial condition:

$$M_{1/2}^a = 2\, M_X , \tag{89}$$

$$m_{H_u}^2 = m_{H_d}^2 = M_X^2 , \tag{90}$$

$$(m_Q^2)_{ij} = c_{Qij}\ \alpha_i \alpha_j M_X^2 ,$$

$$(\mathrm{Q},\ \alpha) \rightarrow (\mathrm{U}, \beta),\ (\mathrm{D}, \gamma),\ (\mathrm{L}, \delta),\ (\mathrm{E}, \epsilon) , \tag{91}$$

$$A_{uij} = -M_X (Y_u)_{ij} + a_{uij}\ \beta_i \alpha_j M_X , \tag{92}$$

$$A_{dij} = -M_X (Y_d)_{ij} + a_{dij}\ \gamma_i \alpha_j M_X , \tag{93}$$

$$A_{eij} = -M_X (Y_e)_{ij} + a_{eij}\ \epsilon_i \delta_j M_X , \tag{94}$$

where M_X was defined as $M_X \equiv |< F_{\tilde{X}} >|/M_5 e^{-kR\pi}$ and we set a natural range of the parameters as $0.1 \lesssim c_{*ij}, a_{*ij} \lesssim 1$. The factor 2 in the right-hand side of (89) comes from the factor $4(g_4^a)^2$ in (42). Since M_{cut} is around 10^{15} GeV, SU(2) and SU(3) couplings of MSSM, $g_4^{a=2}, g_4^{a=3}$, at M_{cut} take the value of 0.7. For simplicity, we fix the normalization of U(1) coupling at M_{cut} as 0.7. Then we obtain the factor 2 in (89) from

$$4(g_4^a)^2[\mu_r = M_{cut}] \simeq 4 \cdot 0.7^2 \simeq 2 .$$

Our aim is to prove that, in our 5D MSSM framework, there exists a mass spectrum that is consistent with the current experimental bounds. We arrange the parameters as

$$c_{*ij},\ a_{*ij} = 1 \quad \text{for } i = j ,$$

$$c_{*ij},\ a_{*ij} = 0.1 \quad \text{for } i \neq j ,$$

to keep the flavor-violating terms as small as possible with a mild hierarchy among the model parameters. Now the model has three free parameters:

$$M_X,\quad \delta_3\ (\sim \delta_2 \sim 3\delta_1),\quad \tan\beta .$$

The KK scale is determined by δ_3 in (41). Since ϵ_3 is smaller than $\mathcal{O}(1)$ from naturalness, (32) leads to the condition:

$$1 \gtrsim \delta_3 \gtrsim m_\tau/v\cos\beta \simeq 0.01/\cos\beta \ . \tag{95}$$

Based on this setup, we calculate mass spectra for various values of $(M_X, \delta_3, \tan\beta)$ and check if they are consistent with the current experimental bounds, in particular, the lower bound of Higgs boson mass. With the flavor-violating soft terms predicted in our model, we then evaluate the rates of the lepton flavor violating processes, i.e. the branching ratios of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays, for each sparticle mass spectrum based on the technology developed in [20] and compared the results with current bounds. In our analysis of MSSM RG equations, we first ignore the off-diagonal terms in (91-94) from the initial condition and numerically solve the MSSM RG equations from M_{cut} to low energies using *Softsusy-3.1.4* [21] with Yukawa off-diagonal terms ignored. After this calculation, we add the off-diagonal terms to give the resultant spectrum.

Below is the list of sample values of $(M_X, \delta_3, \tan\beta)$ that give realistic mass spectra consistent with the bounds on Higgs boson mass and $\mu \to e\gamma$ and $\tau \to \mu\gamma$ branching ratios. We focus on the case with $M_X \leq 600$ GeV because light mass spectra are of more phenomenological interest. The lightest Higgs boson mass m_h, $\mu \to e\gamma$ branching ratio (Br_μ), and $\tau \to \mu\gamma$ branching ratio (Br_τ) of each spectrum are also shown. The results shown here satisfy the current experimental bounds: $m_h > 114.4$ GeV [22], $Br_\mu < 1.2 \times 10^{-11}$ [23] and $Br_\tau < 4.5 \times 10^{-8}$ [24]. The entire mass spectra for three examples are shown in the Appendix. For larger values of δ_3 and/or $\tan\beta$, the spectrum violates the bound on $\mu \to e\gamma$ branching ratio. For smaller M_X and/or $\tan\beta$, the lightest Higgs boson is too light.

M_X (GeV)	500	500	600	600	600
$\tan\beta$	6	10	5	10	15
δ_3	0.06	0.1	0.05	0.1	0.15
m_h (GeV)	115.2	117.5	114.8	118.4	119.1
$Br_\mu \times 10^{12}$	7.6	10	3.9	4.5	8.2
$Br_\tau \times 10^{12}$	4.3	6.1	2.1	2.7	5.7

To $\mu \to e\gamma$ process, loop diagrams containing the following terms contribute:

$$\begin{aligned}
< H_d^0 > (A_e)_{21} &\sim \epsilon_2\delta_1 v_d M_X \sim (\delta_1/\delta_2) m_\mu M_X \sim \frac{1}{3} m_\mu M_X \ , \\
< H_d^0 > (A_e)_{12} &\sim \epsilon_1\delta_2 v_d M_X \sim (\delta_2/\delta_1) m_e M_X \sim 3 m_e M_X \ , \\
(m_L^2)_{12} &\sim \delta_1\delta_2 M_X^2 \sim \frac{1}{3}(\delta_3)^2 M_X^2 \ , \\
(m_E^2)_{12} &\sim \epsilon_1\epsilon_2 M_X^2 \sim \frac{3}{(\delta_3)^2}\frac{m_e}{m_\tau}\frac{m_\mu}{m_\tau} M_X^2 \ .
\end{aligned}$$

The contributions from the terms $(A_e)_{21}$, $(A_e)_{12}$ are almost independent of $\tan\beta$ and δ_3. For the mass spectra listed above, if $(A_e)_{21}$, $(A_e)_{12}$ were the only source of lepton flavor violation, they would give $Br_\mu \sim 7-8\times 10^{-12}$ for $M_X = 500$ GeV and $Br_\mu \sim 3-4\times 10^{-12}$ for $M_X = 600$ GeV. Hence we argue that, for the cases with small $\tan\beta$ and δ_3, the flavor-violating A-terms give dominant contributions. It is obvious that $(A_e)_{21}$ contributes much more strongly than $(A_e)_{12}$. On the other hand, the contribution from the term $(m_L^2)_{12}$ is sensitive to the values of $\tan\beta$ and δ_3, which is roughly proportional to $(\tan\beta)^2$ and $(\delta_3)^4$. (The net value of Br_μ does not reflect this rule because of the interference between $(A_e)_{21}$ contribution and $(m_L^2)_{12}$ contribution.) We thus obtain the upper bounds on $\tan\beta$ and δ_3 when the contribution from the term $(m_L^2)_{12}$ becomes dominant. The contribution from $(m_E^2)_{12}$ is much suppressed by the tiny ratio m_e/m_τ and has negligible impact on $\mu \to e\gamma$ branching ratio.

Here we summarize the features of the sample mass spectra listed above.

(i) The typical SUSY breaking mass scale, M_X, can be as low as 500 GeV and the mass spectrum is within the reach of the LHC.

(ii) The ratio $\delta_3/\tan\beta$ is around 0.01, which means that we need $\epsilon_3 \sim 1$ to have the tau Yukawa coupling. Therefore the 5D superfield of singlet tau is strongly localized towards the IR brane.

(iii) $\mu \to e\gamma$ branching ratio is always higher than O(10^{-12}) and the model can be tested by MEG experiment [25]. in the near future.

(iv) $\tau \to \mu\gamma$ branching ratio is of the same order as $\mu \to e\gamma$ branching ratio.

The feature (ii) originates from the difference between the hierarchy of δ_i's and that of ϵ_i's. The experimental bound on $\mu \to e\gamma$ branching ratio constrains the terms $(m_L^2)_{12}$ and $(m_E^2)_{12}$, which are respectively proportional to $\delta_1\delta_2$ and $\epsilon_1\epsilon_2$, with the same extent. This gives a stronger limit on δ_3 than on ϵ_3 because δ_i's have milder hierarchy. The orders of δ_3 and ϵ_3 are related through

$$\delta_3\epsilon_3/\tan\beta \;\sim\; m_\tau/v \;\simeq\; 0.01 \;,$$

where $\tan\beta$ cannot be smaller than about 4 because otherwise the LEP II Higgs mass bound would not be satisfied. Therefore small δ_3 and large ϵ_3 are favored in this model, which leads to the prediction that $\epsilon_3 \sim 1$.

The feature (iii) results from the existence of the flavor-violating A-term, $(A_e)_{12}$, and the fact that its contribution is independent of $\tan\beta$ and δ_3. The resultant branching ratio is within the future reach of MEG experiment.

The feature (iv) is specific to this model because new physics models normally predict $\tau \to \mu\gamma$ branching ratio larger than $\mu \to e\gamma$ branching ratio as new physics is more likely to affect 3rd generation particles than 1st and 2nd generations. This feature is a consequence

of the feature (ii). Since $\epsilon_3 \sim 1$, SU(2) singlet stau obtains large soft mass through gravity mediation on the IR brane and becomes a few times heavier than SU(2) singlet smuon and selectron if gravity mediation contributes positively as is usually assumed. For $\tau \to \mu\gamma$ process, the term $(A_e)_{32} \sim \epsilon_3 \delta_2 M_X$ always contributes. One can compare its impact on Br_τ with that of $(A_e)_{21}$ on Br_μ by comparing

$$< H_d^0 > (A_e)_{32} \;/\; m_\tau \;\sim\; M_X \quad \text{vs.} \quad < H_d^0 > (A_e)_{21} \;/\; m_\mu \;\sim\; \frac{1}{3} M_X \ .$$

The former is larger by the factor 3. However its effect is canceled by the larger mass of the singlet stau propagating in the loop diagram containing $(A_e)_{32}$. The term $(m_L^2)_{32}$ also contributes when $\tan\beta$ is relatively large. $(m_L^2)_{32}$ is predicted to be about 3 times larger than $(m_L^2)_{12}$. The contribution from $(m_L^2)_{32}$ mainly comes from two types of diagrams, SU(2) singlet smuon propagating in one diagram and singlet stau propagating in the other. However the latter is suppressed by the large stau mass, which partly cancels the effects of large $(m_L^2)_{32}$. The term $(m_E^2)_{23}$ contributes when $\tan\beta$ is relatively large. However, again, its contribution is suppressed by the large mass of the singlet stau propagating in the diagram.

Finally, we discuss the prediction of our model on Δm_K of $K^0 - \bar{K}^0$ mixing and $b \to s\gamma$ branching ratio, based on the paper [26].

The following flavor-violating parameters predicted in our model are relevant to the $K^0 - \bar{K}^0$ mixing:

$$\begin{aligned}
(m_Q^2)_{12} &\sim \alpha_1 \alpha_2 M_X^2 \sim \lambda^5 M_X^2 \;=\; 5 \times 10^{-4}\ M_X^2 \ , \\
(m_D^2)_{12} &\sim \gamma_1 \gamma_2 M_X^2 \sim \frac{1}{\lambda_5} \frac{m_d}{v_d} \frac{m_s}{v_d} M_X^2 \;\simeq\; 3 \times 10^{-5}\ \tan^2\beta\ M_X^2 \ , \\
< H_d^0 > (A_d)_{21} &\sim \gamma_2 \alpha_1 v_d M_X \sim \frac{\alpha_1}{\alpha_2} m_s M_X \;\simeq\; 4 \times 10^{-5}\ M_X^2 \quad \text{for } M_X = 500 \text{ GeV} \ , \\
< H_d^0 > (A_d)_{12} &\sim \gamma_1 \alpha_2 v_d M_X \sim \frac{\alpha_2}{\alpha_1} m_d M_X \;\simeq\; 5 \times 10^{-5}\ M_X^2 \quad \text{for } M_X = 500 \text{ GeV} \ .
\end{aligned}$$

Here we focus on the case with $M_X = 500$ GeV. The average squark mass is around the same scale. Comparing the above predictions with Table 1 in [26], we see that, when $\tan\beta \lesssim 15$, our predictions are below the limits that come from the experimental bound on Δm_K.

For $b \to s\gamma$ process, our model predicts the following flavor-violating parameters:

$$\begin{aligned}
(m_Q^2)_{23} &\sim \alpha_2 \alpha_3 M_X^2 \sim \lambda^2 M_X^2 \;=\; 5 \times 10^{-2}\ M_X^2 \ , \\
(m_D^2)_{23} &\sim \gamma_2 \gamma_3 M_X^2 \sim \frac{1}{\lambda_2} \frac{m_s}{v_d} \frac{m_b}{v_d} M_X^2 \;\simeq\; 2 \times 10^{-4}\ \tan^2\beta\ M_X^2 \ , \\
< H_d^0 > (A_d)_{32} &\sim \gamma_3 \alpha_2 v_d M_X \sim \frac{\alpha_2}{\alpha_3} m_b M_X \;\simeq\; 3 \times 10^{-4}\ M_X^2 \quad \text{for } M_X = 500\text{GeV} \ , \\
< H_d^0 > (A_d)_{23} &\sim \gamma_2 \alpha_3 v_d M_X \sim \frac{\alpha_3}{\alpha_2} m_s M_X \;\simeq\; 4 \times 10^{-3}\ M_X^2 \quad \text{for } M_X = 500\text{GeV} \ .
\end{aligned}$$

Again, we focus on the case with $M_X = 500$ GeV. Comparing the above predictions with Table 6 in [26]. we find that, regardless of $\tan\beta$, our predictions are far below the limits that come from the experimental bound adopted by [26]. Even if we adopt the stronger bound in [27], our predictions are still below the limits.

7 Unusual NLSP and its flavor-violating decay

In this section, we consider NLSP, which is mostly composed of SU(2) singlet charged sleptons. As we found in the previous section, our model favors $\epsilon_3 \sim 1$, i.e. the singlet stau superfield is localized towards the IR brane and receives large gravity mediation effects. This changes the flavor structure of charged slepton mass matrix. We here discuss the model's predictions on the flavor composition of NLSP. In the following, we fix $\epsilon_3 = 1$.

We first review the charged slepton mass matrix. Define

$$\mathcal{A} \equiv A_0 - \mu \tan\beta \ ,$$

where A_0 indicates those parts of A-terms that are proportional to the corresponding Yukawa couplings. We denote RG contributions (gaugino mediation contributions) and D-term contributions to the soft masses of doublet selectron, smuon, stau and singlet selectron, smuon, stau by

$$m_{L1}^2,\ m_{L2}^2,\ m_{L3}^2,\ m_{E1}^2,\ m_{E2}^2,\ m_{E3}^2\ .$$

Since we are focusing on NLSP, whose candidates are singlet sleptons, we neglect relatively small doublet slepton and doublet-singlet mixing terms. Further neglecting terms suppressed by m_e/m_τ, we obtain the following approximate form of the charged slepton mass matrix:

$$m_{\rm slepton}^2 \sim \begin{pmatrix} m_{L1}^2 & 0 & 0 & 0 & 0 & \frac{m_\tau M_X}{3} \\ 0 & m_{L2}^2 & 0 & 0 & \mathcal{A} m_\mu & m_\tau M_X \\ 0 & 0 & m_{L3}^2 & 0 & 0 & \mathcal{A} m_\tau + m_\tau M_X \\ 0 & 0 & 0 & m_{E1}^2 & 0 & 0 \\ 0 & \mathcal{A} m_\mu & 0 & 0 & m_{E2}^2 + (\frac{m_\mu}{m_\tau})^2 M_X^2 & c_{\mu\tau}\ \frac{m_\mu}{m_\tau} M_X^2 \\ \frac{m_\tau M_X}{3} & m_\tau M_X & \mathcal{A} m_\tau + m_\tau M_X & 0 & c_{\mu\tau}\ \frac{m_\mu}{m_\tau} M_X^2 & m_{E3}^2 + M_X^2 \end{pmatrix} . \tag{96}$$

The factor $c_{\mu\tau}$ denotes the coupling of the contact term among singlet smuon, singlet stau and the SUSY breaking sector, and is assumed to be $O(1)$.

In the following, we consider two cases for completeness:

(i) gravity mediation contributions to soft masses are positive, as is usually assumed.

(ii) they are negative.

In case (i), SU(2) singlet stau becomes heavier than singlet smuon and selectron, so that the NLSP will be mainly composed of either singlet smuon or singlet selectron. In order to see which is the one, we consider the mixing mass between singlet smuon and singlet stau, namely $(m_E^2)_{23}$. First note that the difference between m_{E1}^2 and m_{E2}^2 and that between m_{L2}^2 and m_{E2}^2, which arise from RG running, are given by

$$
\begin{aligned}
m_{E1}^2 - m_{E2}^2 &\sim \frac{16}{16\pi^2} \ln\left(\frac{M_{cut}}{M_W}\right) M_X^2 \ (y_\mu)^2 \tan^2\beta \ , \qquad (97)\\
m_{L2}^2 - m_{E2}^2 &\simeq \frac{1}{16\pi^2} \ln\left(\frac{M_{cut}}{M_W}\right) \left(-\frac{18}{5} g_1^2 |M_{1/2}^{a=1}|^2 + \ 6g_2^2 |M_{1/2}^{a=2}|^2 + \frac{9}{5} g_1^2 S\right)\\
&+ \ \{ \ -\frac{1}{2}(\cos^2\theta_W - \sin^2\theta_W) M_Z^2 \ \cos 2\beta \ + \ \sin^2\theta_W \ M_Z^2 \ \cos 2\beta \ \}\\
&\sim \frac{24}{16\pi^2} \ln\left(\frac{M_{cut}}{M_W}\right) \ g_2^6 M_X^2 \ , \qquad (98)
\end{aligned}
$$

where $y_\mu \equiv m_\mu/v$ and $S \equiv m_{Hu}^2 - m_{Hd}^2 + \mathrm{tr}[m_Q^2 - m_L^2 - 2m_u^2 + m_d^2 + m_e^2]$. In deriving (98), we used the relation $M_{1/2}^a \sim 4(g_4^a)^2 M_X$ at the scale M_{cut} shown in (42), and the fact that $M_{1/2}^a/(g_4^a)^2$ is an RG invariant at the 1-loop level.

Singlet smuon mixes with doublet smuon through the term $\mathcal{A} m_\mu$ and with singlet stau through the term $c_{\mu\tau}(m_\mu/m_\tau)M_X^2$. Solving these mixings, the mass eigenstate, which is still dominantly composed of singlet smuon, has mass estimated as

$$
\begin{aligned}
m_{\rm eig}^2 &\simeq m_{E2}^2 + (\frac{m_\mu}{m_\tau})^2 M_X^2 - \frac{2(\mathcal{A} m_\mu)^2}{m_{L2}^2 - m_{E2}^2} - \frac{2(c_{\mu\tau}(m_\mu/m_\tau)M_X^2)^2}{m_{E3}^2 - m_{E2}^2}\\
&\simeq m_{E2}^2 + \left(\frac{m_\mu}{m_\tau}\right)^2 M_X^2 - \frac{2\,(\mu \ \tan\beta \ m_\mu)^2}{(24/16\pi^2)\ln(M_{cut}/M_W) \ g_2^6 M_X^2} - \frac{2(c_{\mu\tau}(m_\mu/m_\tau)M_X^2)^2}{M_X^2}\\
&\simeq m_{E2}^2 + (1 - 2c_{\mu\tau}^2)\left(\frac{m_\mu}{m_\tau}\right)^2 M_X^2 - m_\mu^2 \tan^2\beta / g_2^6 \ . \qquad (99)
\end{aligned}
$$

On the other hand, we can neglect the mixing terms between singlet selectron and other sleptons because they are suppressed by the tiny value of m_e/m_τ.

We now compare the mass of singlet selectron m_{E1}^2 with the mass of the eigenstate, $m_{\rm eig}^2$. Using (97) and (99), we have

$$
\begin{aligned}
m_{\rm eig}^2 - m_{E1}^2 &\simeq -\frac{16}{16\pi^2} \ln\left(\frac{M_{cut}}{M_W}\right) M_X^2 \ (y_\mu)^2 \tan^2\beta + (1 - 2c_{\mu\tau}^2)\left(\frac{m_\mu}{m_\tau}\right)^2 M_X^2 - m_\mu^2 \tan^2\beta / g_2^6\\
&\simeq (1 - 2c_{\mu\tau}^2)\left(\frac{m_\mu}{m_\tau}\right)^2 M_X^2 \ . \qquad (100)
\end{aligned}
$$

In summary, in the determination of the NLSP mass, the effect of the mixing term between the singlet smuon and singlet stau dominates over the Yukawa RG contributions and the effects of

singlet-doublet mixing terms. NLSP is singlet-smuon-like for $c_{\mu\tau} \gtrsim 1/\sqrt{2}$, whereas it is singlet-selectron-like for a relatively small coupling $c_{\mu\tau} \lesssim 1/\sqrt{2}$. Since the factor $c_{\mu\tau}$ affects $\tau \to \mu\gamma$ branching ratio through the term $(m_E^2)_{23} = c_{\mu\tau}(m_\mu/m_\tau)M_X^2$, we have a connection between the value of Br_τ and the flavor of NLSP.

If NLSP is smuon-like, it decays mainly into μ and gravitino. However our model predicts that NLSP can contain a considerable amount of stau component due to the mixing term $(m_E^2)_{23}$, which can be as large as $(m_\mu/m_\tau)M_X^2$. Therefore the branching ratio of a flavor-violating NLSP decay into τ and gravitino can be as large as

$$Br(NLSP \to \tau\ \psi_{3/2}) \sim (m_\mu/m_\tau)^2 \simeq \frac{1}{300}\ . \tag{101}$$

On the other hand, in the context of MFV, the mixing terms of singlet smuon and other sleptons are much smaller as we evaluated in section 5, and such flavor-violating NLSP decays are much suppressed. Thus the flavor-violating decay of NLSP provides a distinct signature of our model. If NLSP is selectron-like, it decays mostly into electron and gravitino. The model predicts that the mixing terms of singlet selectron and other sleptons are suppressed by m_e, although they are much larger than in other models with MFV. Still, as a distinct signature of our model, we expect to observe a rare NLSP decay into τ and gravitino with the branching ratio as large as

$$Br(NLSP \to \tau\ \psi_{3/2}) \sim (m_e/m_\tau)^2\ \simeq\ 10^{-7}\ . \tag{102}$$

Note that the lifetime of NLSP is estimated as

$$t_{NLSP} \simeq 48\pi \frac{|< F_{\tilde{X}} >|^2}{m_{NLSP}^5} \simeq 48\pi \frac{M_X^2 M_{cut}^2}{(m_{NLSP})^5}\ ,$$

which is $\sim 10^{-3}$ sec for $m_{NLSP} = 300$ GeV, $M_X = 500$ GeV and $M_{cut} = 10^{13}$ GeV ($\delta_3 \sim 0.1$). Such a long-lived NLSP, once produced at the LHC, decays outside the detector. There have been interesting proposals [28] for the way to trap charged NLSPs outside the detector. Detailed studies of the NLSP decay can allow us not only to measure the gravitino mass and the four-dimensional Planck mass but also to test the flavor-structure of NLSP predicted in our model.

In case (ii), the gravity mediated contributions are negative, so that SU(2) singlet stau is the dominant NLSP component. Although this singlet-stau-like NLSP is as usual in SUSY models with gravitino LSP, we again expect to observe a rare decay of NLSP as a signature of the model. The branching ratio of NLSP decay into μ and gravitino is predicted to be as large as

$$Br(NLSP \to \mu\ \psi_{3/2}) \sim (m_\mu/m_\tau)^2\ \simeq\ \frac{1}{300}\ . \tag{103}$$

8 Conclusion

We have investigated a simple 5D extension of MSSM in RS spacetime, where 5D geometry controls both the SUSY breaking mediation mechanism and the Yukawa coupling hierarchy. The Yukawa coupling hierarchy is naturally explained by the localization of matter superfields in the 5D bulk. SUSY breaking effects arise from two sources: contact terms between the SUSY breaking sector and the MSSM fields (gravity mediation), and RG effects (gaugino mediation). The former are flavor-violating and the latter flavor-conserving. Using the experimental data on fermion masses and mixings, we have determined the 5D disposition of matter superfields and calculated SUSY breaking mass spectra including flavor-violating terms. We have numerically checked that our framework can give a realistic mass spectrum consistent with all the experimental constraints.

We have estimated the flavor-violating effects induced by RG running in the context of MFV and compared them with those from gravity mediation predicted by the model. We have proved that our model provides a different pattern of flavor-violating terms, namely, flavor violation of A-terms and SU(2) singlet soft masses can be much larger than the MFV case.

Our model has several distinct predictions. First, the $\mu \to e\gamma$ branching ratio is larger than $O(10^{-12})$, regardless of $\tan\beta$ and the seesaw scale, for sparticle masses $\lesssim$ 2 TeV. This originates from the basic structure of our model, namely, the hierarchy of Yukawa couplings and the gravity mediation contributions to A-terms are rooted on the same 5D disposition of the matter superfields. Hence, using the experimental data on the charged SM fermion masses, CKM matrix and the neutrino oscillation parameters, we can fix the orders of flavor-violating A-terms. Second, $\tau \to \mu\gamma$ branching ratio may not be larger than $\mu \to e\gamma$ branching ratio. This is because our model predicts IR-localized singlet stau superfield and it gains an additional soft mass through gravity mediation on the IR brane. Third, since RS geometry warps down the effective cutoff scale, which can be characterized by the seesaw scale, gravitino is LSP and the dark matter candidate. Forth, NLSP is likely to be either smuon-like or selectron-like because the gravity mediation contribution pushes up the singlet stau mass. Furthermore our model predicts flavor-violating NLSP decays with the rates much higher than those expected in the MFV case.

The Yukawa coupling hierarchy is one of the long-standing problems in the Standard Model. The 5D MSSM on Randall-Sundrum background offers a solution to this problem from the geometrical point of view. In this model, flavor-violating soft SUSY breaking terms have the same geometrical origin as the Yukawa coupling hierarchy. Therefore, the flavor structure among sparticles can be a clue to understand the origin of flavors among the SM fermions, even if the origin lies at an energy scale far above the electroweak scale. Gravitino is always LSP due

to the warped geometry, and NLSP is predicted to be dominantly composed of SU(2) singlet sleptons and long-lived. At collider experiments, in this case, supersymmetric events can be fully reconstructed without missing energy, which allows us not only to identify the dominant flavor content of NLSP, but also to measure the rates of flavor-violating decays of sparticles if it is sizable. Thus, our framework can be tested in the future.

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Appendix: Sample Mass Spectra

M_X	500	600	600
$\tan\beta$	6	5	15
δ	0.06	0.05	0.15
h^0	115.2	114.7	119.1
H_0	1217	1452	1423
H_{A0}	1215	1450	1420
$H^\pm$	1218	1452	1422
$\tilde{g}$	1845	2172	2270
χ_1^0	568	693	647
χ_2^0	814	984	986
χ_3^0	927	1090	1133
χ_4^0	964	1131	1156
$\chi_1^\pm$	815	986	987
$\chi_2^\pm$	967	1135	1163
$\tilde{u}_L$	1616	1893	2004
$\tilde{d}_L$	1618	1895	2005
$\tilde{u}_R$	1542	1804	1912
$\tilde{d}_R$	1533	1793	1900
$\tilde{t}_1$	1288	1517	1604
$\tilde{t}_2$	1582	1848	1932
$\tilde{b}_1$	1529	1790	1878
$\tilde{b}_2$	1552	1821	1911
$\tilde{e}_L$	578	685	714
$\tilde{e}_R$	338	401	414
$\tilde{\nu}_e$	572	681	709
$\tilde{\tau}_1$	574	683	688
$\tilde{\tau}_2$	611	730	735
$\tilde{\nu}_\tau$	571	680	700
$Br_\mu \times 10^{12}$	7.6	3.9	8.2
$Br_\tau \times 10^{12}$	4.3	2.1	5.7

Table 1: Particle mass spectra for three samples for different values of $(M_X, \tan\beta, \delta)$.

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